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Contents

1 Introduction

1.1 Historical overview of iceberg hazards

The primary example of hazards associated with icebergs is obviously the sinking of the *RMS Titanic* on 14-15 April 1912 on her maiden voyage en route from Southampton, UK to New York, US after colliding with an iceberg that may have been as high as about 60m above sea level (Bartlett, 2011).

However, this was not the first accident with a ship colliding with an iceberg, nor was it the last. The earliest reported collision took place in the Hudson Strait in 1686, when the sailing ship *Happy Return* sank.

Observations of icebergs have for over a century been recorded and reported to marine vessels. In fact, on 12 April the *RMS Titanic* herself forwarded a telegram from the *Amerika* that large icebergs had been sighted about 20km to the south of the position where she later sank (Anon., 1912).

Despite the vast technological advances that have taken place during the past century, collisions with icebergs still occur. On June 17, 2004, the trawler *Sólborg* was 37 kilometres out of Newfoundland's Conception Bay when she collided with an iceberg in foggy conditions, despite the crews' attempts to avoid this by aiding navigation with data from an on-board radar (Curtis, 2006).

1.2 Present status of iceberg threat

Presently, the International Ice Patrol (IIP), operated by the US Coast Guard funded by 13 countries, collects iceberg data from a variety of observational platforms, including ships, designated flights and satellite-born instruments. The IIP and the Canadian Ice Service issue a daily iceberg analysis for the Grand Banks region off Newfoundland. The iceberg analysis is published in text bulletins and a graphical chart.

Icebergs are also encountered in other Arctic regions which are the subject of hydrocarbon exploration activity. As no routine iceberg warning service exists for these regions, it is dependent on the operation companies to ensure that they have sufficient information to evaluate and prepare for the iceberg threat.

1.3 Observations of icebergs

The standards for iceberg reporting have been defined by the Canadian Ice Service (CIS) and International Iceberg Patrol (IIP). This is mainly for historical regions, the IIP was set up in 1913 in the aftermath of the *Titanic* disaster, and authorities in other areas where icebergs occur such as the seas around Greenland and Antarctica, and in the Russian and Norwegian Arctic have followed these conventions.

The CIS Manual of Ice (MANICE) (CIS, 2005) provides tables for the size and shape of icebergs, and for their coding in telex reports. The sizes include a number of terms specific to iceberg, such as 'growler' and 'bergy bit' (see Table 1.1), and can be used to estimate the mass of the iceberg.

The iceberg can also be reported by its shape (Table 1.2). Tabular icebergs are typical of the Antarctic, and only a few glaciers in the Arctic produce them. Arctic icebergs are typically pinnacled or blocky.

Reliance on radar satellite remote sensing using synthetic aperture radar (SAR) imaging sensors has reduced the requirement for direct visual observations from ships and aircraft. These, and radar observations from ships, are described in Section 2.

Description	*Illustration*	*Height*	*Length*	*Code*	*Estimated Weight (Megatons)*
Growler		< 1 m	< 5 m	1	0.001
Bergy Bit		1 - < 5 m	5 - < 15 m	2	> 0.01
Small Iceberg		5 – 15 m	15 – 60 m	3	> 0.1
Medium Iceberg		16 – 45 m	61 – 120 m	4	2.0
Large Iceberg		46 – 75 m	121 – 200 m	5	10.0
Very Large Iceberg		> 75 m	> 200 m	6	> 10.0
Not Specified		-	-	7	
Radar Target		-	-	X	

Table 1.1: Iceberg sizes (S_i), from CIS and IIP.

Description	*Code*	*Photograph*	*Average height to draft ratio*
Tabular	1		1:5
Non-Tabular	2		1:5
Domed	3		1:4
Pinnacled	4		1:2
Wedged	5		1:5
Drydocked	6		1:1
Blocky	7		1:5
Ice Island	8		
Not Specified	0		
Undetermined (Radar)	X		

Table 1.2: Iceberg shapes (S_h), from CIS and IIP.

1.4 Modelling of icebergs

In order to forecast iceberg trajectories it is necessary to have access to a model that describes the iceberg movements from basic knowledge of the system. The main factors that determine the iceberg drift is the strength and direction of wind, currents, waves and ice drift (if present). If these factors, as well as the iceberg geometry, are known one should be able to forecast iceberg movements. For an operational case at sea, such measurements may conceivably be available and a simple code for estimating trajectory for certain input parameters are the basis of the iceberg drift model from the Canadian Hydraulic Centre (CHC). However, most often these factors are not observed and one needs to rely on various models for obtaining the above mentioned variables (or forcing fields). If the forcing is accurately known, a reliable iceberg trajectory forecast can be produced. Unfortunately, usually either the iceberg geometry or the forcing fields (or, in most cases, both) are not known to the desired degree and the state of iceberg trajectory forecasting is still in a development stage. However, given the threat that icebergs pose on many operations at sea, even relatively inaccurate trajectory forecast can be useful for planning operations in areas with icebergs.

The basic dynamical equations for modelling of iceberg trajectories and iceberg deterioration are given in Section 3. Section 4 describes remaining challenges for iceberg trajectory modelling. Section 5 finally gives recommendations for future work.

2 Remote detection of icebergs

This describes the principal sensors and techniques used for the remote detection of icebergs from satellites, and from ships. We exclude here airborne and ground based visual observations.

Whist very large Antarctic sensors can be tracked by low resolution active microwave sensors (scatterometers), those commonly found in the Arctic and of interest to oil and gas activities are typically medium to small. These require high resolution satellite imaging fort heir detection, such as that provided by active microwave synthetic aperture radar (SAR). SAR has been available since the early 1990's, with the European Space Agency (ESA) and Canadian Space Agency (CSA) the principal providers. ESA's SAR missions, ERS-1/2 and Envisat, have been primarily science-based and it was difficult to obtain frequent routine coverage of a required area. The new ESA SAR satellite launched in 2014, Sentinel-1a, is designed for operational use and will provide the routine acquisitions that are needed. The CSA Radarsat-1 and -2 satellites have been operational since the mid-1990's and have provided routine operational coverage, typically the waters of the Canadian Archipelago and east coast down to Newfoundland. Since 2008 this coverage has been extended to the European Arctic and particularly the waters around Svalbard. Radarsat-1 and -2 SAR data has been used for operational detection by the Danish Meteorological Institute (DMI) for their Greenland Ice Service, and products from this are now part of the MyOcean project portfolio (ref: SEAICE_ARC_SEAICE_L4_NRT_OBSERVATIONS_011_007).

SAR sees the Earth's surface as a return signal (backscatter) that is proportional to the surface roughness. Differences in roughness allows the different surface types to be determined. Icebergs are typically very rugged, and provide a high level of backscatter. The sea surface surrounding them can vary from flat calm, when icebergs and other objects such as ships are very obvious, to very rough with waves making it very difficult to detect objects. The addition of sea ice further complicates the ability to spot iceberg objects.

The primary image processing technique used for iceberg detection is the Constant False Alarm Rate (CFAR) concept.

2.1 Constant False Alarm Rate (CFAR)

CFAR evaluates the SAR backscatter at a location against the surrounding background backscatter. If the location is found to differ significantly from the background, it is classified as a target. Issues arise when the SAR image also includes objects, such as sea ice and ships, that are not icebergs.

A description of how to calculate the CFAR statistic is in Section 10.2 pp.279-82 of Oliver and Quegan, 2004. Here it is defined as:

$$\frac{\bar{I}_T/\bar{I}_B-1}{\sqrt{V_B}}>t$$

where $\bar{I}_T \equiv \sum_{j=1}^{m} I_j/m$ is the average target intensity over a central region of interest (ROI) of *m* pixels. The ROI is surrounded by a guard ring of pixels to prevent leakage from the target into the boundary ring of *M* pixels, which is used to calculate a background level $\bar{I}_B$, as well as the background variance V_B.

The calculation of the CFAR statistic uses a raw, not despeckled, image. This is because it is assumed noise in the image could be targets. A typical example for Radarsat-2 ScanSAR Wide is a target ROI of 3×3 pixels (150×150 metres), surrounded by a guard ring of 20 pixels (1,000 metres), and an outer background ring of 5 pixels (250 metres) (Figure 2.1).

A threshold value is then applied to the CFAR image. Values below the threshold are taken to be open water. Values above the threshold are assumed to be icebergs. The level at which the threshold is set determines how many false detections occur (the false alarm rate). The aim is to minimise the number of false alarms. This typically runs into a number of issues:

1. Lack of ground truth. There are too few iceberg observations to verify if all the detections in an image are correct. Use of Automatic Identification System (AIS) data helps to remove positive detections that are ships.
2. Lack of information on iceberg topography and radar interaction. Whilst co-polarisation (HH or VV) SAR has been widely used, the recent availability of cross-polarisation (HV and VH) , or fully polarimetric (HH+VV+HV+VH), SAR has yet to be properly evaluated. Lack of detailed surveys of icebergs limits the ability to test how different configurations affect detection, and also limit progress in forecast model development.

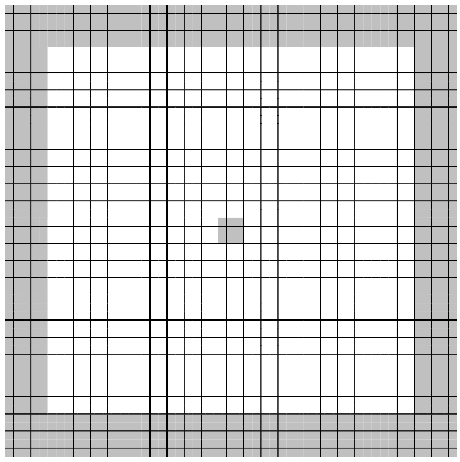

Figure 2.1: Diagram of target detection configuration showing ROI for a target in the centre, a guard ring to avoid leakage from the target into the background, and a boundary ring to estimate the background level.

2.2 Shipboard Radar Detection

For ships operating within iceberg infested waters, onboard radar is the primary method of detection. This provides a more oblique and limited view of the surface than a satellite platform, but has the advantage of being always available.

Detection capability with shipboard radars is dependent on:

1. Radar frequency
2. Height of radar installation

   As the radar is line-of-sight, small ships have a poorer view than larger ships. There is also the issue of placement and ensuring that masts and superstructure do no block the radar view in some directions.
3. Operator skill

   Use of shipboard radar for good quality iceberg observations requires a level of skill on the part of the operator. Digital processing systems available on some radars can assist with this, but the operator has to keep track of the targets to gain any insight into how they may be moving. Typically, an iceberg has to be sighted visually to confirm the radar observation as to its size and shape.

2.3 Validation of Remote Detection

Validation of remote detection requires independent observations of icebergs, preferably with tracking over a number of days. There is a significant lack of observation data coincident with modern SAR acquisitions. Such data that is collected tends to be related to oil and gas exploration activity and was not available to this project. Therefore it has not been possible to run a thorough validation of satellite CFAR or shipboard radar detection.

3 Systems for iceberg trajectory forecasting

In this section we will describe the basis of iceberg trajectory forecasting. We start with a description of the equations that describe iceberg dynamics and we discuss i) the momentum balance that determines the drift speed and direction, and ii) the mass balance. The latter is an important factor for modelling of iceberg trajectories for long times, or for studies on climatology of icebergs. After the initial section, we discuss three different iceberg drift models, namely the iceberg drift and deterioration model from CHC, followed by a description of the model developed by the Nansen Environmental and Remote Sensing Centre (NERSC), and finally the model at Norwegian Meteorological Institute (MET Norway) is discussed.

3.1 General framework for iceberg forecasting

3.1.1 Equations for iceberg drift

The basis for prediction or evaluating the movement of icebergs is to outline a momentum balance for icebergs. The forces acting on an iceberg comes from i) wind, ii) the iceberg velocity relative to water, iii) the Coriolis force, iv) forces from surface waves impinging on the iceberg, v) pressure (gravitational) forces in case the sea surface has a tilt, and vi) forces from movements relative to sea ice. Furthermore generation of internal waves can also be important for the momentum budget (Pite et al., 1995), but this process is not considered here.

The momentum balance of an iceberg can thus be written as

$$m\frac{d\mathbf{u}_B}{dt} = \mathbf{F}_A + \mathbf{F}_W + \mathbf{F}_C + \mathbf{F}_r + \mathbf{F}_P + \mathbf{F}_{SI} \quad , \qquad (1)$$

where, say, $m=(M_{berg}+m_{added})\approx 1.5M_{berg}$ is the mass of the iceberg and the (added) mass of water that the iceberg drags along its movements (here taken to be the same for all types of icebergs), $\mathbf{u}_B=(u_B,v_B)$ is the iceberg velocity in the $\mathbf{x}=(x,y)$ direction, $\mathbf{F}_A$ is the air drag, $\mathbf{F}_W$ is the water drag, $\mathbf{F}_C$ is the Coriolis force, $\mathbf{F}_r$ is the stress from surface gravity waves (often called radiation stress), $\mathbf{F}_P$ is the effective force due to pressure gradients in the upper ocean (e.g., due to the sloping sea surface), and $\mathbf{F}_{SI}$ is the force due the iceberg-sea ice interaction (we thus neglect the generation of internal waves).

The wind drag is formulated as

$$\mathbf{F}_A = \frac{1}{2}\rho_A C_A A_A \left|\mathbf{U}_A\right| \mathbf{U}_A \quad , \qquad (2)$$

where ρ_A is the density of air, C_A is a non-dimensional drag coefficient, A_A is the cross section area (or sail area), and U_A is the wind speed; index A indicates air-related quantities (here we assume that the speed of the iceberg is negligible as compared to the wind speed). As a note, it is considered as common knowledge that the ice will move with 2-8% of the wind speed, and having a deflection of 15-20° to the right of the wind direction due to the rotation of the earth.

For water drag the force on the iceberg is

$$\mathbf{F}_W = \frac{1}{2}\rho_W C_W \sum_k A_W(k) \left|\mathbf{U}_W(k) - \mathbf{u}_B\right| \left(\mathbf{U}_W(k) - \mathbf{u}_B\right) \quad , \qquad (3)$$

where index W relates to water properties, and k is the vertical levels (i.e, the model is based on that A_W is known at different depth).

The Coriolis force is given by

$$\mathbf{F}_C = mf\mathbf{k} \times \mathbf{u}_B \quad , \tag{4}$$

where *f* is the Coriolis parameter. The force from the pressure gradient in the water is

$$\mathbf{F}_P = -mg\nabla_H \zeta \tag{5}$$

where *g* is the acceleration due to gravity and $\nabla_H\zeta$ is the tilt of the sea surface ζ. The tilt of the sea surface is not easily observed but can be taken from an ocean circulation model. It should be noted in this context that the horizontal pressure gradient that acts on an iceberg is not only a result of the surface tilt, as it also depends on the horizontal gradient of the atmospheric sea level pressure.

The radiation force from waves is described by (Longuet-Higgins, 1977; Savage, 2007)

$$\mathbf{F}_r = \frac{1}{2}\rho_w C_{wd} g L H_w^2 \mathbf{k}_w \quad , \tag{6}$$

where C_{wd}=0.3 is the wave force coefficient, and H_w is the wave height[1], finally $\mathbf{k}_w$ is the direction of the waves.

For the ice force on an iceberg Lichey and Hellmer (2001) gave following expression

$$\mathbf{F}_{SI} = \begin{cases} 0, & A_{SI} \le 15\%, \\ \frac{1}{2}\rho_{SI} C_{SI} A_{SI} \left|\mathbf{u}_{SI} - \mathbf{u}_B\right|\left(\mathbf{u}_{SI} - \mathbf{u}_B\right), & 15\% \le A_{SI} \le 90\%, \\ \infty\left|\mathbf{u}_{SI} - \mathbf{u}_B\right|\left(\mathbf{u}_{SI} - \mathbf{u}_B\right), & 90\% \le A_{SI} \text{ and } P \ge P_{SI}, \end{cases} \tag{7}$$

where $\mathbf{u}_{SI}$ is the sea ice velocity, A_{SI} is the sea ice coverage, *P* is the stress on the iceberg from the ice and P_{SI} reflects the value for which the ice resist the forces acting on the iceberg without failing (Lichey & Hellmer, 2001; Savage, 2008). The constant C_{SI} depends on the ice strength, or the effective ice failure pressure, and the CHC model is based on the description of C_{SI} given by Savage (2008). The formulation in Eq. (7) for high ice concentration (A_{SI}≥90%) essentially implies that the iceberg is frozen in the ice and will move with the ice speed[2].

It should be noted that a standard explicit numerical scheme cannot be used for iceberg trajectory modelling due to the inclusion of the Coriolis force, and either higher order schemes or implicit schemes has to be used (Kubat et al., 2005; Savage, 2001). For the CHC model the standard time step is taken to be 2 minutes (Sayed, pers, comm.). The model is started with zero initial velocity of the icebergs. Although not perfect this will not affect the forecast significantly on timescales longer than 1 hour.

3.1.2 Equation for iceberg deterioration

The deterioration of icebergs is probably not very important for short term forecasting of icebergs in cold areas. However, for climatological studies, or when tracking specific icebergs for a long time, the

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1Wave height is twice the wave amplitude; the wave model gives significant wave height (H_S), which is related to the wave height as a_w= H_S /1.421573 (Savage, 2007). Note that this expression is very similar to *a*= H_S /, as would be given by standard derivation of significant wave height and wave height (Komen et al., 1994; Phillips, 1977)

2This is also the way that ”frozen in” icebergs are treated numerically in the model. The icebergs are advected with the speed of the sea ice.

deterioration of icebergs will become important (here it is possible to have a service where a combination of modelling and tracking an iceberg will enable following the same iceberg over time). It should be noted that the iceberg geometry (both above and below the water surface) is an important parameter in the momentum balance for the iceberg. The iceberg geometry will to some extent influence the direction and speed of the iceberg, thus, it is possible to adjust (assimilate) the iceberg geometry to fit observations, if an iceberg trajectory is known. This may improve the forecast of an iceberg trajectory (Gusdal and Broström, 2012).

The deterioration of icebergs is more complex to describe than its motion. The deterioration will depend on the geometry of the iceberg and how it interacts with the surrounding elements. This interaction is complex and the present formulation is based on empirical relations. The most important processes, and their relative magnitudes, are listed below (Savage, 2001)

1. Wave induced erosion (say 60 %).
2. Wave induced calving (say 20%).
3. Forced convection in water (say 15%).
4. Solar radiation (say 3.5%).
5. Buoyant convection in water, wind convection (say 1.5%).

For large icebergs, cracking of icebergs by wave bending may also be important (Squire, 2007; Squire et al., 1995; Wadhams, 2000); this is not included in this study. The readers are also referred to other studies for a more detailed description of the deterioration processes (Kubat et al., 2007; Savage, 2001).

We can write the mass balance schematically as

$$\frac{dM_{berg}}{dt} = -V_m^{w-e} - V_m^{focrced-conv} - V_m^{bouyant-conv} - V_m^{solar} - V^{calving} \tag{8}$$

where M_{berg} is the mass of the ice berg, V_m^{w-e} is wave erosion, $V_m^{focrced-conv}$ is melting from forced convection (i.e., movements of the iceberg through water and $V_m^{bouyant-conv}$ is melting from buoyant convection. V_m^{solar} is melting from solar forcing and $V^{calving}$ is mass loss from calving from the iceberg. This latter process may create new smaller icebergs, but as of today it is difficulty to make accurate predictions of this. Nevertheless, a probability function of finding smaller icebergs in the vicinity of a larger iceberg may very well be considered.

3.2 Iceberg forecasting system at the Canadian Hydraulics Centre

The above formulations for iceberg momentum and mass balances are quite general and some features still needs to be specified. Here we use the model from CHC to exemplify various parameterizations. One should notice that the model developed by CHC has to some extent been developed for operational use at sea and thus carries some features that focus on easily observed variables rather than on mathematical distinct definitions of the different forces. Given the overall uncertainty in forcing data, the simplifications/formulations used in the CHC model is probably not severe for practical applications.

3.2.1 Dynamical model

The iceberg geometry

One key feature of the CHC iceberg model is that it was primarily developed for pinnacle icebergs (characteristic for the Grand Banks area) and uses an empirical relation for the area of an pinnacle iceberg (the sail height and depth of the iceberg) both above and below the water surface. The sail area, A_A, is given by

$$A_A = a_0 L + b_0 , \qquad (9)$$

where L is waterline length of the iceberg, and a_0=28.194 m, b_0=-1420.2 m^2 are empirical constants (Barker et al., 2004; Kubat et al., 2005). In case the expression gives negative value of sail height (i.e, for L smaller than about 50 m) the sail area is set to zero. The area beneath the water surface, A_W, is in a similar way given by

$$A_W(k) = a_k L + b_k , \qquad (10)$$

where a_k, b_k are empirical constants determined at every 10 m interval in the deep (Barker et al., 2004; Kubat et al., 2005).

The pressure gradient

The CHC iceberg model has been developed for use in an operational setting at sea, and is therefore based on easily observed quantities. The pressure gradient is not easily observed: however, as it is more or less linked to the water motions and in practice the geostrophic motion for open ocean conditions, we may derive the surface tilt from observations on the ocean currents. The pressure gradient can be estimated as (Savage, 2001)

$$\mathbf{F}_P = m\left(\frac{d\overline{\mathbf{U}_W}}{dt} + f\mathbf{k}\times\overline{\mathbf{U}_W}\right) ,$$

where $\overline{\mathbf{U}_W}$ is a weighted water current between the sea surface and the keel depth. Note that this formulation is different from more standard formulations based on the pressure gradient from a sloping sea surface (Savage, 2001). In the present CHC model a simpler formulation is used such that

$$\mathbf{F}_P = mf\mathbf{k}\times\overline{\mathbf{U}_W} , \qquad (11)$$

where

$$\overline{\mathbf{U}_W} \approx \frac{\sum_k A_W(k)\mathbf{U}_W(k)}{\sum_k A_W(k)} . \qquad (12)$$

The wave forcing in the CHC model is split into swell and wind waves where the wind waves are in the direction of the wind, i.e., $\mathbf{k}_w = \mathbf{U}_A / |\mathbf{U}_A|$, while the swell must be given a direction either from observations or from a model, this reflects the easily observed forcing variables for a real case at sea. However, from a forecasting point of view a more distinct formulation can be made.

3.2.2 Deterioration model

One important factor in the model is the relation between iceberg mass and waterline length. The relation is

$$M_{berg} = \kappa \rho_i L^3 \quad , \tag{13}$$

where ρ_i is ice density. The parameter κ has a value of approximately 0.45 (Barker et al., 2004). Based on iceberg observations both from Arctic and Antarctic Research Institute (AARI) and Ice Data Acquisition Program (IDAP) studies, this seem to be somewhat high and value around 0.35 may be more appropriate for Barents Sea (K. Johannessen, Personal communication). The deterioration will be expressed as a velocity representing the rate of change of the waterline length of the iceberg, the total loss of mass needs to be calculated using Eq. (7). The model assumes a self-similar shape of icebergs such that the relation between iceberg mass and iceberg geometry is solely determined by e.g. iceberg length (although it should be mentioned that there exist different iceberg types that use different self-similar shapes).

The formulas below involve coefficients evaluated to give the melt rates in units of m/s. The units of those coefficients are not listed here, as customary done in similar literature (e.g. White et al. 1980, and Savage 2001).

Surface melting due to solar radiation

The melting velocity for solar radiation *I* is given by (Savage, 2001)

$$V_s = \frac{I}{\Gamma \rho_i}(1-\alpha) \quad , \tag{14}$$

where Γ is the latent heat of melting of ice (334 kJ/kg), and α is the albedo. The values of the albedo range from 0.1 for clear ice surfaces to 0.95 for fresh snow: α=0.7 is frequently used. The model is not very sensitive to the exact value of this parameter and the CHC model use a constant solar radiation of 203.5 Wm^{-2}.

Melting due to buoyant vertical convection

An iceberg will affect the density of the water in the vicinity of the iceberg. Accordingly, density driven currents will be induced that affect the melting of the iceberg. The dynamical feature of this process is complex and the following empirical correlation is used to estimate the melt rate (Neshyba & Josberger, 1980)

$$V_b = 2.78(\Delta T) + 0.47(\Delta T)^2 \quad , \tag{14}$$

where ΔT is the difference between the far field water temperature, T_∞, and the freezing point temperature, T_{fp}; i.e. $\Delta T = T_\infty - T_{fp}$. Note that to convert V_b to units of m/s, Eq. (14) should be divided by 31,536,000 (Kubat et al., 2007).

The equation for the freezing point to be used in the model is

$$T_{fp} = T_f(S)\, e^{-0.19\left(T_\infty - T_f(S)\right)} \quad , \tag{15}$$

where T_f is the sea water freezing temperature based on the far field salinity, *S*. The sea water freezing temperature depends on the salinity, *S*, according to (Løset, 1993)

$$T_f(S) = -0.036 - 0.0499\,S - 0.000112\,S^2 \quad . \tag{16}$$

which is a good approximation for 1.77% ≤ S ≤ 3.5%. However, in the CHC code, the value of T_f is -1.86 °C.

Forced convection

The relative velocity between the iceberg and water current contributes to the process of melting the keel (the relative velocity between the iceberg and the water is important to replace the cold water close to the iceberg with warm sea water). Also wind can contribute to melting of the sail and this is described in a similar way as the melting in water. The surface melt due to forced convection can be expressed as

$$V_f = \frac{q_f}{\rho_i\,\Gamma} \quad , \tag{17}$$

where q_f is the heat flux,

$$q_f = Nu\,k_f\,\Delta T / L \quad , \tag{18}$$

where k_f is the thermal diffusivity of the fluid (air or sea water). The Nusset number, Nu is given by

$$Nu = C\,Re^{0.8}\,Pr^{0.4} \quad , \tag{19}$$

where C=0.058, and the Reynolds number, Re, and Prandl number, Pr, are defined as

$$Re = V_r\,L/\nu \quad , \tag{20}$$

and

$$Pr = \nu / k_f \quad . \tag{21}$$

Here V_r is the relative velocity between the iceberg and the fluid and ν is the kinematic viscosity. We note that in calculations of the drag forces, which are used in modelling the drift, the variation of water current with depth is taken into account (Kubat et al., 2005). However, for the above calculation of the relative velocity, V_r, a mean current was considered to give adequate accuracy and is consistent with overall formulation that is based on observations.

Wave erosion

This is a major source of iceberg deterioration. White et al. (1980) developed the following equation to estimate the melt rate of a notch at the waterline

$$V_{we} = 0.000146\left(\frac{R}{H_w}\right)^{0.2}\left(\frac{H_w}{\tau}\right)\Delta T \quad , \tag{22}$$

where V_{we} is the melt rate in m/s, R is the roughness height of the ice surface, typically 0.01m (White et al., 1980), ΔT is the temperature difference between the sea water and the iceberg (defined below Eq. (14)), and τ and H_w are the wave period and wave height in units of seconds and meters, respectively. The melt rate V_{we} can be up to 1 m/day for a temperature difference of 1K. We note that ΔT must be known with high accuracy to evaluate melting in a reliable way.

Calving

Calving can be caused by several mechanisms but the most important is the breaking of overhanging slabs of ice (Savage, 2001). A notch at the waterline usually forms due to wave erosion. As the erosion progresses, the notch deepens and size of the ice hanging above the notch increases. At a certain stage, the bending stresses cause fracture of the ice, and the overhanging slab collapses. The model representation of this process (White et al., 1980) can be summarized as follows: the critical length of an overhanging slab at which fracture (calving) occurs, F_l, is given by

$$F_l = 0.33\left(37.5 H_w + h^2\right)^{1/2} , \qquad (23)$$

where H_w is the wave height and h is the thickness of the overhanging slab (both in meters). The expression for the overhanging slab thickness can be expressed as (Savage, 1999)

$$h = 0.196 L . \qquad (24)$$

For steady wave action, the calving interval, t_c, is given by

$$t_c = F_l / V_{we} . \qquad (25)$$

Savage (1999) also carried out an analysis of the shape of the overhanging ice and obtained the following expression for the calved ice volume,

$$\overline{V_c} = 0.64\, L\, F_l\, h . \qquad (26)$$

The above relations were validated using available estimates of observed calved ice masses (Savage 2001). Aside from the mechanism discussed here, calving can occur due to fracture caused by internal stresses or overturning. Such mechanisms appear to have a minor contribution to calving (Savage, 2001). They are also too complex to include in the present simple mechanical model.

3.3 Iceberg forecasting system at the Nansen Environmental and Remote Sensing Centre

The iceberg forecasting system at the Nansen Environmental and Remote Sensing Centre (NERSC) is described by Keghouche et al. (2009) (see also Keghouche, 2010; Keghouche et al., 2010). It is derived based on Bigg et al. (1997), and mainly follows the framework which is outlined in 2.1.1 above. Nevertheless, there are some differences that are worth pointing out.

In the NERSC model, the mass that is accelerated is set to the mass of the iceberg, i.e., no accompanying water is considered (m_{added}=0). Note however than when results from the NERSC model are compared with observed iceberg drift, three different specifications of the iceberg mass is used as input, and the results may thus implicitly also represent inclusion of accompanying water.

Next, the formulation of the wind drag is somewhat more advanced in the NERSC model:

$$\mathbf{F}_A = \left(\frac{1}{2}\rho_A C_A A_{VA} + \frac{1}{2}\rho_A C_{DA} A_{HA}\right)\left|\mathbf{U}_A - \mathbf{u}_B\right|\left(\mathbf{U}_A - \mathbf{u}_B\right) \qquad (27)$$

where C_A and C_{DA} are the form and skin drag coefficients, respectively. A_{VA} is the vertical cross-sectional area of the iceberg above the sea level, while A_{HA} is the horizontal area of the iceberg in contact with the atmosphere.

For the pressure gradient force, the NERSC model use (Keghouche et al. 2009):

$$\mathbf{F}_P = -M_B \mathbf{g} \sin(\alpha) \tag{28}$$

where **g** is the acceleration due to gravity and α is the angular tilt of the sea surface. However, taken as is this formulation is obviously wrong, since this force will act in the vertical direction. A correct representation would be to replace **g** sin(α) by $g \nabla_H \zeta$ where ζ is the sea level as given by the ocean circulation model from which the results for ocean currents is taken, thus reproducing Eq. (5).

The radiation stress from surface gravity waves ($\mathbf{F}_r$) is not explicitly included in the NERSC model. Instead the action from sea swell is disregarded and it is assumed that the action of wind waves is included parametrically in the wind drag. This is achieved by fitting the model results to observed trajectories while varying the atmospheric drag coefficient C_A.

The formulation of the wind drag in the NERSC model is thus rather odd: the small error due to neglecting water velocity in Eq. (2) is corrected, but what we suspect are much larger misrepresentations, invoking a parameterization of the action of wind waves by means of a modified wind drag coefficient, and disregarding swell, are introduced. Keghouche et al. (2009) state that the form of the radiation stress as specified by Bigg et al. (1997) is proportional to the wind drag, but this is not correct. A similar aspect of wind drag and wave radiation in the momentum balance equation in Bigg et al. is that the radiation stress from the waves is taken to act in the wind direction. However, in their parameterization, Bigg et al. has a wave radiation term that is proportional to the square of the wave amplitude, and the wave amplitude is proportional to the square of the wind speed. Hence, the wave radiation term becomes proportional to $\mathbf{U}_A^4$ while the wind drag is proportional to $\mathbf{U}_A^2$. This is a significant discrepancy, given that the drag coefficients in the NERSC model are determined by data fitting with observations of more than 2 months' trajectories, and the resulting wind drag varies by more than an order of magnitude during this observation period (their Fig. 5).

Finally we note that the original version of the NERSC model (Keghouche et al., 2009) does not include a mass budget equation, so that the iceberg mass is taken to be constant throughout the time period that is spanned by the trajectory simulations. Subsequently, a mass budget equation was added to the dynamic model (Keghouche et al., 2010). Their mass budget equation is similar to Eq. (9) above, but melting due to solar radiation and erosion in the shape of calving are disregarded. Moreover, wave erosion is parameterized by an alternative formula due to Gladstone and Bigg (2001), in which the rate of deterioration is given by the Beaufort scale magnitude, the water temperature and the sea ice concentration. Thus, the wave height enters implicitly as depending on the Beaufort magnitude, analogously to the treatment of form drag due to waves in the NERSC model implementation of the iceberg momentum equation. We also register that the parameterization by Gladstone and Bigg is an extrapolation of results from very low wind speeds, so its validity for rough sea states can be questioned.

3.4 Iceberg forecasting system at MET Norway

The iceberg drift and deterioration model at MET Norway was developed and implemented in 2009. The implementation at MET Norway is based on the CHC model, and was undertaken in collaboration with CHC. The implementation has been documented and validated by comparison of drift results to observations from the Barents Sea by Broström et al. (2009b). Furthermore, the quality of the forcing fields and how this impacts trajectory results was documented from a hindcast study by Broström et al. (2009a).

Gusdal and Broström (2012) performed iceberg drift experiments with the MET Norway model with varying descriptions of the iceberg size and shape and related parameterizations (drag coefficients). Their study revealed that forecasted trajectories are sensitive to this description, and as discussed in Section 4 here, this is a result which may be exploited in future work with improvements of e.g. the MET Norway model.

Model input was taken from results from hindcasts for atmospheric circulation, waves and ocean circulation. It was also possible to apply results from operational models as forcing, thus producing truly operational trajectory forecasts. With respect to model input and user interface, the MET Norway model was adapted to the existing local infrastructure at the time. Furthermore the implementation, particularly the user interface and the output formatting, was tailored to accommodate for the requests by the funding organization, Statoil.

The MET Norway model has not been included among the in-house emergency services, since there has not yet been funding available neither for such an operational service (externally or internally), nor for maintenance of this model. Meanwhile, the format of the files produced by MET Norway which provide results with which the iceberg drift and deterioration model is forced has been changed. Moreover, the general framework of the user interface provided by MET Norway for external users of our emergency services has been completely overhauled during the time that has passed from the implementation of the iceberg drift and deterioration model. As a consequence of the lack of funding for maintenance, the iceberg drift and deterioration model has not been ported to the new user interface framework.

4 Challenges in iceberg trajectory forecasting

Iceberg drift modeling is a complex task that includes a variety of ingredients which all have a significant impact on the quality of the results. Errors, or even inaccuracies, in any of these aspects can severely degrade the quality. We can generally divide the "ingredients" into three categories: [1] iceberg properties (size, shape, mass) and how they change, [2] environmental forcing (ocean currents, sea ice, wind, waves), and [3] parameterization of how iceberg momentum is generated by these forcing mechanisms.

4.1 Uncertainties in iceberg properties

In order to describe the motion of an iceberg precisely, an accurate description of the iceberg volume, its density relative to the density of the surrounding waters, and the distribution of its mass (the shape of the iceberg) are needed. If the volume and relative density are known, the ratio of the iceberg volume which is above and below sea level, respectively, can be calculated. It is necessary, but not sufficient, to know this ratio in order for the wind drag and drag by ocean currents to be calculated correctly. Note that if the ratio is known, the volume of the submerged part of the iceberg is known when the volume of the part of the iceberg that is above sea level is known.

Moreover, the shape of the iceberg must be known in order for the form drag due to the vertical profile of the wind and ocean current to be calculated correctly. The shape of the iceberg above sea level can be specified e.g. from photography, and the shape of the submerged part may in principle be derived from other instrumentation (possibly from sonar data). In an operational setting, we will rarely have access to such data. At best, a reliable categorical description of the iceberg is available, from which an approximation of the shape of the submerged part of the iceberg can be derived from its shape above sea level. Categories that are presently implemented in iceberg trajectory models are among the following: [i] tabular, [ii] tilted tabular, [iii] weathered, [iv] pinnacle, [v] blocky, [vi] bergy bit, [vii] growler (Spring, 1994).

In order to produce an accurate trajectory forecast, changes in the mass and shape must also be forecasted. Usually, a mass budget equation is solved such as Eq. (8) above. This equation uses a number of parameterizations in order to describe the various processes that contribute to the ultimate demise of an iceberg. It is particularly difficult, if not impossible, to include a reliable description of the disintegration of an iceberg due to calving (of bergy bits etc.). This binary process must be described probabilistically based on empirical data, but it is the actual time of calving that has an impact on the trajectory. Finally, we are not aware of any implementation of iceberg shape forecasts except implicitly by transition from one iceberg category to another.

None of the aspects of iceberg properties listed above will be known in detail in a realistic operational setting for trajectory forecasting. Hence, imperfect descriptions of all of these aspects of iceberg properties introduce uncertainties in the resulting trajectory forecasts. We speculate that the shape of the submerged iceberg and the parameterizations of source terms in the mass budget equation are the two main sources of uncertainty in the context of iceberg properties.

4.2 Uncertainties in forcing fields

It is well known that weather forecasting for synoptic scales (100-1000 km) is usually reliable for lead times of 2-7 days, somewhat dependent on the general conditions. However, two aspects of weather forecasting may have a significant negative impact on the quality of the forecasts that are relevant for iceberg drift. First, the observational density is low in the Arctic, making the analysis from which the forecast is integrated more uncertain than e.g. over continental Europe or North America. Second, sub-synoptic circulation features which can emerge in the Arctic, such as polar lows, are difficult to forecast accurately. Polar lows are intense low pressure systems with winds of gale or storm strength, and should such phenomena be neglected in the environmental forcing the accuracy of iceberg drift forecasting will become severely degraded.

The quality of wave forecasts generally depends strongly on the quality of the forecasted winds. Thus, the evaluation of weather forecasting for iceberg drift given above is also relevant for wave forcing in iceberg drift forecasts. This is certainly the case for locally generated waves (wind waves), but to a somewhat lesser degree the case for swell. For the large distances over which swell propagate, the wave dynamics is well known and wave forecasts are accurate (Komen et al., 1994).

However, one aspect of wave propagation that may be highly relevant for iceberg drift is either poorly described by wave models, or not described at all. This aspect is wave propagation in ice infested waters. Waves will be dampened by sea ice, with shorter waves attenuating more swiftly than longer waves (Wadhams 2000, Broström and Christensen, 2008). Hence, wave forecasts are expected to be less accurate in the marginal ice zone (MIZ) than in the open ocean and this may have a negative effect on the quality of drift forecasts for icebergs in the MIZ.

Icebergs that drift in waters which are infested by sea ice will have a trajectory which will be significantly affected by the drift of the sea ice. Sea ice drift forecasts are available from coupled ocean – sea ice models. The quality of ice drift forecasts is generally poorly known. However, in the EU Marine Core Service project MyOcean the accuracy of ice drift forecasts have been monitored since January 2012. Validation results (presently available from http://myocean.met.no/ARC-MFC/V2Validation/timeSeriesResults/). We note that the validation metrics reveal a moderate deterioration of the forecast quality as the lead time increases. At the time of writing (June 2014), the RMS of the forecasted sea ice drift distance increase from 7 km for the first day to 11 km for the tenth day.

As we have seen so far, inaccuracies that must be expected in the environmental forcing are expected to be reflected in the quality of iceberg drift forecasting. Nevertheless, the forcing field which has the largest impact on iceberg drift, ocean currents (Kubat et al., 2007) is yet to be discussed. Efforts in modelling the large scale ocean circulation may benefit from assimilation of altimeter data for sea level anomalies. Tidal motion is deterministically forced by celestial motion, but its representation in the forcing fields will depend on the accuracy in the description of the bottom topography. However, in large regions momentum is dominated by meso-scale motion associated with fronts, meanders and eddies. Such motion is generally not observed with a sufficient resolution for initialization of ocean circulation forecasts. As indicated by Melsom et al. (2012), it is presently difficult to capture observed surface drift even when employing an ensemble forecast for the ocean circulation.

Finally, we note that in shallow seas icebergs may become beached. It is a challenging task to correctly describe and forecast iceberg beaching. First, the size and shape of the iceberg, particularly sub-surface, needs to be modelled with a very high accuracy. Second, the bottom topography must be known on a highly detailed level, and this is not always the case. Finally, processes such as iceberg melt and, more relevant in the context of environmental forcing, buoyant lifting by tides, must be forecasted with very high quality in order for on-set of post-beaching drift to be modelled correctly.

4.3 Uncertainties in iceberg model dynamics

One large unknown for iceberg trajectory modelling is the iceberg geometry, e.g., the cross section area at different depths. Given the veering of the wind driven current (i.e., the Ekman spiral) the depth of the iceberg is an important parameter for the direction of the drift. For deep icebergs, the lower part may well stick down into a region where the current has a different drift direction than the surface current. If an iceberg is detected and tracked a crude description of the iceberg geometry may be derived by adjusting the iceberg geometry in the model until a good fit with data is obtained. This will of course not be precise, and we may well reach a situation where a false forcing can be compensated by a false iceberg geometry. Nevertheless, by adjusting the iceberg geometry in the model toward the observed trajectory, it is likely that the forecast accuracy improves (Gusdal and Broström, 2012).

One aspect of the forcing that remains somewhat uncertain, and is to some extent coupled to uncertainties in the geometry, is the orientation of the iceberg with the wind, currents and waves. In search and rescue (SAR) modelling it is well known that objects at sea drift at a certain angle to the wind, it can either be to the left or the right of the wind direction, and the angle can switch during the drift (although in most cases it is quite stable) (Breivik and Allen, 2008). The reason for such behaviour is possibly that the object orients itself with the wind and its uneven geometry forces the wind to blow faster at one side than on the other side, creating a pressure force toward the side with the fast wind. If this is important for icebergs remains to be evaluated and some theoretical development to quantify the effects is needed.

Another uncertainty that can be connected to iceberg modelling is the possibility that an iceberg that moves through a stratified ocean will generate internal waves. This will manifest itself as a drag force: the process is well known and is referred to as dead water (Gill, 1982). The possibility for this to be an important factor depends on the stratification and the iceberg geometry. In any case, it may well be an important factor for describing iceberg trajectories, but the effect remains to be tested and outlined for icebergs.

5 Recommendations

As pointed out in Section 3 above, model results for the ocean currents have a quality which is moderate at best, and this is a main concern in iceberg trajectory forecasting. In fact, although Keghouche et al. (2009) have access to trajectory data from 17 icebergs over a three year period, they discard all of these data but four trajectories from one season. They adopt this quite selective approach since “the observed icebergs were mostly driven by […] mesoscale activity, […] not being represented in the model”. So the accuracy of the remaining model trajectories would presumably be significantly restricted due to lack of ocean current observations and/or the limited resolution in the ocean circulation model. Such an approach is justifiable when the iceberg drift model is examined from the point of view as a stand-alone model. However, in an operational setting the luxury of selecting cases based on the state of the ocean circulation must be discarded.

From the evaluation of uncertainties in Section 3, and their likely effect of limiting the quality of iceberg trajectory forecasts, we see two approaches that have the potential for adding relevant information about the trajectory projection. The first approach is to implement a probabilistic description of the main sources for uncertainty, and the second is to take advantage of the trajectory observations that have been made prior to the initialization of the trajectory forecast, if such data are available.

Based on the evaluation of the present trajectory forecast models, we suggest that the main sources for uncertainty are [a] the forcing by ocean currents, [b] the initial size and shape of the iceberg, and [c] wave erosion. We expect that the latter quantity is mostly relevant for forecasts that are made for a drift period of about one week or more, i.e., in cases where a significant modification of the initial shape and size of the iceberg may occur.

Melsom et al. (2012) evaluated results from an advanced 100-member ocean ensemble forecast system for applications in search and rescue operations. Based on trajectory simulations by results from each ensemble member from a known initial position, a convex hull that spanned all trajectories was identified. This approach was validated using data from GPS tracking of drifting buoys in the North Atlantic Ocean, the Nordic Seas and the Barents Sea. Melsom et al. found that for one-day drift the size of the convex hulls varied by an order of magnitude. The observed trajectory was more frequently found inside the hull when this was a large region. Hence, the ensemble variability was deemed to be of modest use in search and rescue operations, and any adaptation in iceberg trajectory forecasting should be evaluated carefully before it is implemented in an operational service. Note however that Melsom et al. found that the ensemble mean trajectory forecast was an improvement of the alternative deterministic approach.

An alternative to the advanced system considered by Melsom et al. (2012) is to implement a simplified model which is tailored to describe circulation in the ocean mixed layer (such models are known as reduced gravity models). An ensemble simulation may then be produced quickly, since a reduced gravity model is much less computationally intensive than a full 3D ocean circulation model. The challenge will be how best to perturb such a simplified model. We suggest that the potential vorticity is perturbed, with amplitudes based on results from decadal simulations with a proper horizontal resolution. Then, if the iceberg’s trajectory history is known, a probabilistic forecast may be produced by weighting trajectories from each member based on the various members’ success in hindcasting the iceberg trajectory.

Trajectory observations may also be exploited in traditional, deterministic forecast. In this case, the discrepancy of a hindcast simulation may be applied as an anomaly in the model results. Such a modification has a potential to significantly improve trajectory forecasts at least inside a time window of a few days, due to the persistence of mesoscale ocean circulation features.

Regarding the next item (item [b]), the dependence of drift trajectories on the initial size and shape of the iceberg, observations are expected to be of moderate or low quality. Assuming that the environmental forcing agents (current, wave, wind etc.) are known, we may then generate an ensemble simulation based on a probability distribution of the iceberg’s size and shape. The spread in this probability distribution should be adapted to the quality and level of detail in the available information of the iceberg shape and size. In a situation where ensemble results for ocean currents are available, this iceberg property ensemble simulation will be an ensemble of ensembles. Results from previous, well-observed cases will then be important in order to extract reliable information from a vast set of trajectories. Gusdal and Broström (2012) found that changes in the trajectory speed from adjustments in the initial size and shape was limited to about 20%.

All of the terms in the iceberg deterioration equation (Eq. (9)) are based on parameterizations which renders the description of the terms subject to significant uncertainties. The relative importance of the processes that lead to iceberg deterioration depends on the conditions of the iceberg and its physical environment. Regarding the final item (item [c]), wave erosion is a leading term in most situations (Savage, 2001; Kubat et al., 2007) (wave induced calving is next in importance, and this also depends strongly on wave induced erosion). An exception is icebergs that are in waters with a sea ice cover that causes wind waves to become substantially attenuated. The main concern regarding the representation of wave erosion is not the description of the wave field, but the unknown quality of the parameterisation of wave erosion. For forecasts that span weeks or months it may therefore be worth considering how modifications in the parametric representation of wave erosion affect the resulting iceberg trajectory.

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